LAPLACE TRANSFORM

AM-III / KUNAL NAVLAKHI

Definition:-

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s)$$

TYPE I:- FORMULA METHOD:

$f(t)$	$L[f(t)]$
1	$1/s$
e^{at}	$\frac{1}{s-a}$
e^{-at}	$\frac{1}{s+a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
t^n	$\frac{\overline{\lvert n+1}}{s^{n+1}}$

DERIVATIONS:-

(1) Let $f(t) = e^{at}$

$$\therefore L[f(t)] = \int_0^\infty e^{-st} \cdot e^{at}\,dt = \int_0^\infty e^{-t(s-a)}\,dt = \left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty$$

$$\therefore L[e^{at}] = \frac{1}{s-a}$$

Similarly $L[e^{-at}] = \frac{1}{s+a}$

When $a = 0$;

$$L[1] = \frac{1}{s}$$

In general $L[k] = \frac{k}{s}$; k = constant

(2) $f(t) = \sin at$

$$\therefore L[f(t)] = \int_0^\infty e^{-st} \sin at \, dt$$

$$= \left[\frac{e^{-st}}{s^2+a^2}\left(-s\sin at - a\cos at\right)\right]_0^\infty$$

$$\therefore L[\sin at] = \frac{a}{s^2+a^2}$$

(3) $f(t) = \cos at$

$$L[f(t)] = \int_0^\infty e^{-st} \cos at \, dt$$

$$= \left[\frac{e^{-st}}{s^2+a^2}\left(-s\cos at + a\sin at\right)\right]_0^\infty$$

$$\therefore L[\cos at] = \frac{s}{s^2+a^2}$$

(4) $f(t) = \sinh at$

$$\therefore L[f(t)] = \int_0^\infty e^{-st} \sinh at \, dt$$

$$= \int_0^\infty e^{-st}\left(\frac{e^{at}-e^{-at}}{2}\right)dt \quad \left\{\because \sinh at = \frac{e^{at}-e^{-at}}{2}\right\}$$

$$= \frac{1}{2}\left[\int_0^\infty e^{-st} \cdot e^{at} dt - \int_0^\infty e^{-st} \cdot e^{-at} dt\right]$$

$$= \frac{1}{2}\left\{\int_0^\infty e^{-t(s-a)} dt - \int_0^\infty e^{-t(s+a)} dt\right\}$$

$$= \frac{1}{2}\left\{\left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty - \left[\frac{e^{-t(s+a)}}{-(s+a)}\right]_0^\infty\right\}$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \frac{1}{2}\left[\frac{2a}{s^2-a^2}\right]$$

$$\therefore L[\sinh at] = \frac{a}{s^2-a^2}$$

(5) $f(t) = \cosh at$

$$\therefore L[f(t)] = \int_0^\infty e^{-st} \cosh at \, dt$$

$$= \int_0^\infty e^{-st}\left(\frac{e^{at}+e^{-at}}{2}\right)dt \quad \left\{\because \cosh at = \frac{e^{at}+e^{-at}}{2}\right\}$$

$$\therefore L[f(t)] = \frac{1}{2}\left[\int_0^\infty e^{-st}e^{at}dt + \int_0^\infty e^{-st}.e^{-at}dt\right]$$

$$= \frac{1}{2}\left[\int_0^\infty e^{-t(s-a)}dt + \int_0^\infty e^{-t(s+a)}dt\right]$$

$$= \frac{1}{2}\left\{\left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty + \left[\frac{e^{-t(s+a)}}{-(s+a)}\right]_0^\infty\right\}$$

$$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \frac{1}{2}\left[\frac{2s}{s^2-a^2}\right]$$

$$\therefore L[\cosh at] = \frac{s}{s^2-a^2}.$$

(6) $f(t) = t^n$

$$\therefore L[f(t)] = \int_0^\infty e^{-st}.t^n dt$$

Let $st = u$
$\therefore dt = \frac{du}{s}$

t	0	∞
u	0	∞

$$\therefore L[f(t)] = \int_0^\infty e^{-u}.\left(\frac{u}{s}\right)^n.\frac{du}{s}$$

$$= \frac{1}{s^{n+1}}\int_0^\infty e^{-u}.u^n du$$

$$\therefore L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}} = \frac{n!}{s^{n+1}} \quad \{\text{if } n \in I\}$$

IMPORTANT PROPERTIES OF LAPLACE TRANSFORM:-

(1) Linear Transform:

$$L[a_1f_1(t) + a_2f_2(t)] = a_1L[f_1(t)] + a_2L[f_2(t)]$$

Proof: L.H.S. $= L[a_1f_1(t) + a_2f_2(t)]$

$$= \int_0^\infty e^{-st}[a_1f_1(t) + a_2f_2(t)]dt$$

$$= a_1\int_0^\infty e^{-st}f_1(t)dt + a_2\int_0^\infty e^{-st}f_2(t)dt$$

$$= a_1L[f_1(t)] + a_2L[f_2(t)]$$

$\therefore$ L.H.S. = R.H.S.

TYPE II :- First Shifting Theorem :

i.e. $L[e^{-at} f(t)] = \bar{f}(s+a)$; where $L[f(t)] = \bar{f}(s)$

Proof:

$$L.H.S. = L[e^{-at} f(t)]$$

$$= \int_0^\infty e^{-st} [e^{-at} f(t)] dt$$

$$= \int_0^\infty e^{-(s+a)t} f(t) dt$$

$$= \bar{f}(s+a)$$

$\therefore$ L.H.S. = R.H.S.

TYPE III :- Second Shifting Theorem:

i.e. $L[F(t)] = e^{-as} \bar{f}(s)$ where $F(t) = \begin{cases} f(t-a) ; & t > a \\ 0 ; & t < a \end{cases}$

& $L[f(t)] = \bar{f}(s)$

Proof:

$$L.H.S. = L[F(t)]$$

$$= \int_0^\infty e^{-st} F(t) dt$$

$$= \int_0^a e^{-at} F(t) dt + \int_a^\infty e^{-st} F(t) dt$$

$$= 0 + \int_a^\infty e^{-st} f(t-a) dt \; ; \text{ Let } t-a=u \quad \begin{array}{|c|c|c|} \hline t & a & \infty \\ \hline u & 0 & \infty \\ \hline \end{array}$$

$\therefore dt = du$

$$= \int_0^\infty e^{-s(u+a)} f(u) du$$

$$= e^{-as} \int_0^\infty e^{-su} f(u) du$$

$$= e^{-as} \bar{f}(s)$$

$\therefore$ L.H.S. = R.H.S.

TYPE IV :- $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \bar{f}(s)$ where $L[f(t)] = \bar{f}(s)$

By Definition, $\bar{f}(s) = \int_0^\infty e^{-st} f(t) dt$

Diff: w.r.t. s on both sides,

$$\therefore \frac{d}{ds}\bar{f}(s) = \frac{d}{ds}\int_0^\infty e^{-st} f(t)\,dt$$

By D.U.I.S.

$$\frac{d}{ds}\bar{f}(s) = \int_0^\infty \left[\frac{\partial}{\partial s} e^{-st} f(t)\right] dt = \int_0^\infty \left[f(t)\cdot\frac{\partial}{\partial s}\cdot e^{-st}\right] dt$$

$$= \int_0^\infty f(t)(-t.e^{-st})\,dt = -\int_0^\infty e^{-st}\left[t f(t)\right] dt$$

$$\therefore \frac{d}{ds}\bar{f}(s) = -L\left[t f(t)\right]$$

$$\therefore L\left[t f(t)\right] = (-1)\frac{d}{ds}\bar{f}(s)$$

Similarly, $L\left[t^2 f(t)\right] = (-1)^2 \frac{d^2}{ds^2}\bar{f}(s)$

$$L\left[t^n f(t)\right] = (-1)^n \frac{d^n}{ds^n}\bar{f}(s)$$

$$\therefore L.H.S. = R.H.S.$$

TYPE V :- $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}(s)\,ds$ where $L[f(t)] = \bar{f}(s)$ & $\lim_{t\to 0^+}\frac{f(t)}{t}$ exists

$$R.H.S. = \int_s^\infty \bar{f}(s)\,ds$$

$$= \int_s^\infty \left[\int_0^\infty e^{-st} f(t)\,dt\right] ds \qquad \left\{\because \bar{f}(s) = \int_0^\infty e^{-st} f(t)\,dt\right\}$$

Changing order of integration, since limits are constants,

$$R.H.S. = \int_{t=0}^\infty \left[\int_s^\infty e^{-st}\,ds\right] f(t)\,dt$$

$$= \int_0^\infty \left[\frac{e^{-st}}{-t}\right]_s^\infty f(t)\,dt = \int_0^\infty \frac{e^{-st}}{t}\cdot f(t)\,dt$$

$$= \int_0^\infty e^{-st}\left[\frac{f(t)}{t}\right] dt = L\left[\frac{f(t)}{t}\right]$$

$$\therefore R.H.S. = L.H.S.$$

LAPLACE TRANSFORM

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TYPE I :- FORMULA METHOD:

Find the Laplace transform of the following-

(1) $t^2 - e^{-2t} + e^t$ (2) $\frac{t}{4} - \frac{\sin t}{3} + \frac{\sin 2t}{24}$ (3) $(t^2+1)^2$

(4) $(\sin 2t - \cos 2t)^2$ (5) $a\cos^2 2bt$ (6) $\cosh^2 4t$ (7) $\sin^3 t$

(8) $\sin(\omega t + \alpha)$ (9) $\cos t . \cos 2t$ (10) $t^2 - 3t + 5$ (11) $t^{5/2}$

(12) $t + t^2 + t^3$ (13) $\sin^3 2t$ (14) $\sin 2t . \cos 3t$ (15) $\sin t . \cos t$

(16) $\cos^2 t$ (17) $\sin 2t . \sin 3t$ (18) $t^{-1/2}$ (19) $\cos(\omega t + \alpha)$

(20) $\sin\sqrt{t}$ (21) $\frac{\cos\sqrt{t}}{\sqrt{t}}$

TYPE II :- Using First Shifting Theorem

i.e. $L[e^{-at} f(t)] = \bar{f}(s+a)$ where $L[f(t)] = \bar{f}(s)$.

(22) $e^{-bt}\cos at$ (23) $t^2 e^{3t}$ (24) $e^{at}(2\cos bt - 3\sin bt)$ (25) $t^n e^{-at}$

(26) $(t+1)^2 e^t$ (27) $\cos at \sinh at$ (28) $2e^t \sin 4t \cos 2t$

(29) $e^{4t} . t^{3/2}$ (30) $e^{-t}\sin^2 t$ (31) $e^{4t}\cosh 5t$ (32) $e^{-3t} t^{-1/2}$

(33) $e^{-t}\cosh t$ (34) $\cosh at . \sin at$ (35) $e^t \sin t . \cos t$ (36) $e^{-at} t^n$

(37) $\sinh at \sin at$ (38) $(\cos 2t + \frac{1}{2}\sin 2t)e^t$ (39) $e^{-2t} . t^3$

(40) $2te^{2t}$ (41) $t^2 e^t$ (42) $4t^3 e^{-2t}$ (43) $\frac{1}{2} t^4 e^{-3t}$ (44) $e^t \cos t$

(45) $3e^{2t}\sin 2t$ (46) $5e^{-2t}\cos 3t$ (47) $4e^{-5t}\sin t$ (48) $2e^t \sin^2 t$

(49) $\frac{1}{2} e^{3t}\cos^2 t$ (50) $e^t \sinh t$ (51) $3e^{2t}\cosh 4t$ (52) $2e^{-t}\sinh 3t$

(53) $\frac{1}{4} e^{-3t}\cosh 2t$ (54) $2e^t(\cos 3t - 3\sin 3t)$ (55) $3e^{2t}(\sinh 2t - 2\cosh 2t)$

TYPE III :- Using Second Shifting Theorem

i.e. $L[F(t)] = e^{-as}\bar{f}(s)$ where $F(t) = \begin{cases} f(t-a) ; & t > a \\ 0 ; & t < a \end{cases}$

& $L[f(t)] = \bar{f}(s)$

(56) $F(t) = \begin{cases} (t-1)^3 ; & t > 1 \\ 0 ; & 0 < t < 1 \end{cases}$

(57) $f(t) = \begin{cases} \cos(t-\alpha) ; & t > \alpha \\ 0 ; & t < \alpha \end{cases}$

(58) By using fundamental definition, find the Laplace transform of $f(t)$ where:

(i) $f(t) = \begin{cases} a & ; 0<t<b \\ 0 & ; t>b \end{cases}$

(ii) $f(t) = \begin{cases} t & ; 0<t<4 \\ 5 & ; t>4 \end{cases}$

(iii) $f(t) = \begin{cases} 0 & ; 0<t<1 \\ t & ; 1<t<2 \\ 0 & ; t>2 \end{cases}$

(iv) $f(t) = \begin{cases} \sin 2t & ; 0<t<\pi \\ 0 & ; t>\pi \end{cases}$

(v) $f(t) = \begin{cases} (t-1)^2 & ; t>1 \\ 0 & ; 0<t<1 \end{cases}$

TYPE IV :- Using $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \bar{f}(s)$

where $L[f(t)] = \bar{f}(s)$

(59) $\frac{t}{2a} \sinh at$ (60) $t^2 \cos at$ (61) $t(2\sin 3t - 3\cos 3t)$

(62) $t^2 \cos kt$ (63) $t e^{3t} \sin 2t$ (64) $t^2 \sin at$ (65) $t e^t f(t)$

TYPE V :- Using $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}(s)\,ds$ where $L[f(t)] = \bar{f}(s)$ & $\lim_{t\to 0^+} \frac{f(t)}{t}$ exists

(66) Find Laplace transform of $\sin at$ & hence show that $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$

(67) $\frac{1}{t}(1-\cos at)$ (68) $\frac{1}{t}(e^{at} - e^{bt})$ (69) $\frac{1}{t}(\cos at - \cos bt)$

(70) $\frac{\sinh t}{t}$ (71) $t^{-1} e^{-t} \sin t$ (72) $\frac{\sin^2 t}{t}$ (73) $\frac{1}{t^2}(1-\cos t)$

(74) $\frac{1}{t}(1-e^t)$ (75) $\frac{1}{t}(e^{-at} - e^{-bt})$ (76) $\frac{\sin 2t}{t}$ (77) $\frac{\sin^3 t}{t}$

(78) Find Laplace transform of $\frac{\sin t}{t}$ & hence show that $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$.

TYPE VI :- Using $L[f(at)] = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$ where $L[f(t)] = \bar{f}(s)$

(79) If $L[f(t)] = \frac{8+12s-2s^2}{(s^2+4)^2}$; find $L[f(2t)]$

(80) If $L[f(t)] = \frac{1}{s}e^{-1/s}$, find $L[e^{-t}f(3t)]$

(81) If $L[\text{erf}\sqrt{t}] = \frac{1}{s\sqrt{s+1}}$, find $L[t\,\text{erf}(2\sqrt{t})]$

(82) If $L[f(t)] = \frac{s^2-s+1}{(2s+1)^2\cdot(s-1)}$, find $L[f(2t)]$

TYPE VII :- Using Convolution theorem:

i.e. $\bar{f}_1(s)\cdot\bar{f}_2(s) = \int_0^t f_1(u)f_2(t-u)\,du$

where $\bar{f}_1(s) = L[f_1(t)]$ & $\bar{f}_2(s) = L[f_2(t)]$

(83) Verify convolution theorem for $f_1(t) = t$ & $f_2(t) = e^{at}$

(84) Verify convolution theorem for $f_1(t) = t^2$ & $f_2(t) = e^{-at}$

TYPE VIII :- Using Laplace Transform of an Integral:

i.e. $L\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\bar{f}(s)$ where $L[f(t)] = \bar{f}(s)$

(85) Verify $L\left[\int_0^t u^2e^{-u}\,du\right] = \frac{1}{s}L[t^2e^{-t}]$

(86) $\int_0^t e^t\frac{\sin t}{t}\,dt$ (87) $\int_0^t x\cosh x\,dx$ (88) $\cosh t\int_0^t e^x\cosh x\,dx$

(89) $\int_0^t e^u\cdot u^3\,du$ (90) $\int_0^t e^u\cos u\,du$ (91) $\int_0^t \frac{1-e^{-x}}{x}\,dx$

(92) $t\int_0^t e^{-4x}\sin 3x\,dx$ (93) $e^{-3t}\int_0^t t\sin 3t\,dt$ (94) $\int_0^t t\cdot e^{-4t}\sin 2t\,dt$

TYPE IX :- Using Laplace Transform of Derivatives:

i.e. $L[f'(t)] = s\bar{f}(s) - f(0)$ where $L[f(t)] = \bar{f}(s)$

(95) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 5y$; given $y(0) = 2$ & $y'(0) = -4$.

(96) Given $L\left[2\sqrt{\frac{t}{\pi}}\right] = \frac{1}{s^{3/2}}$; show that $L\left[\frac{1}{\sqrt{\pi t}}\right] = \frac{1}{\sqrt{s}}$

(97) $\frac{d}{dt}\left(\frac{\sin t}{t}\right)$

(98) Given $f(t) = t+1 \; ; \; 0<t<2$
$\quad\quad = 3 \; ; \; t>2$

find $L[f(t)]$ & $L[f'(t)]$

TYPE X :- Initial Value Theorem:

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} s\bar{f}(s) \text{ where } L[f(t)] = \bar{f}(s)$$

(99) Verify initial value theorem for the voltage function $(5+2\cos 3t)V$; also state its initial value.

(100) Verify initial value theorem for

(i) $3-4\sin t$

(ii) $(t-4)^2$

Also state their initial values.

~~(101)~~ TYPE XI :- Final Value Theorem:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s.\bar{f}(s) \text{ where } L[f(t)] = \bar{f}(s)$$

(101) Verify final value theorem for

(i) $2+3e^{-2t}\sin 4t$

(ii) $4+e^{-2t}(\sin t+\cos t)$

Also state their final values.

MISCELLANEOUS:-

(102) Given $L[J_0(t)] = \frac{1}{\sqrt{1+s^2}}$, show that

(i) $L[t\,J_0(at)] = \frac{s}{(s^2+a^2)^{3/2}}$ (ii) $L[e^{-at}J_0(at)] = \frac{1}{\sqrt{s^2+2as+2a^2}}$

(iii) $\int_0^\infty J_0(t)dt = 1$ (iv) $\int_0^\infty t e^{-3t} J_0(4t) = \frac{3}{125}$

(103) Evaluate:-

(i) $\int_0^\infty t^3 e^{-t}\sin t\,dt$ (ii) $\int_0^\infty e^{-2t}\sin^3 t\,dt$ (iii) $\int_0^\infty t e^{-3t}\sin t\,dt$

(iv) $\int_0^\infty e^{-2t} \cdot \frac{\sinh t}{t} dt$. (v) $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt$

(vi) $\int_0^\infty t e^{-3t} \sin t \, dt$ (vii) $\int_0^\infty e^{-3t} \cos^3 t \, dt$

(viii) $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$ (ix) $\int_0^\infty e^{-t} \frac{\sin t}{t} dt$

(x) $\int_0^\infty e^{-2t} \frac{\sinh t \cdot \sin t}{t} dt$.

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ANSWERS

(1) $\dfrac{3s^3+2s^2+2s-4}{s^3(s+2)(s-1)}$ (2) $\dfrac{1}{s^2(s^2+1)(s^2+4)}$ (3) $\dfrac{s^4+4s^2+24}{s^5}$ (4) $\dfrac{1}{s}-\dfrac{4}{s^2+16}$

(5) $\dfrac{a}{2s}+\dfrac{as}{2s^2+32b^2}$ (6) $\dfrac{1}{2}\left[\dfrac{1}{s}+\dfrac{s}{s^2-64}\right]$ (7) $\dfrac{6}{(s^2+1)(s^2+9)}$ (8) $\dfrac{s\sin\alpha+\omega\cos\alpha}{s^2+\omega^2}$

(9) $\dfrac{s(s^2+5)}{(s^2+1)(s^2+9)}$ (10) $\dfrac{5s^2-3s+2}{s^3}$ (11) $\dfrac{15}{8}\sqrt{\dfrac{\pi}{s^7}}$ (12) $\dfrac{1}{s^2}+\dfrac{2}{s^3}+\dfrac{6}{s^4}$ (13) $\dfrac{48}{(s^2+4)(s^2+36)}$

(14) $\dfrac{1}{2}\left(\dfrac{4s^2-20}{(s^2+1)(s^2+25)}\right)$ (15) $\dfrac{1}{s^2+4}$ (16) $\dfrac{s^2+2}{s(s^2+4)}$ (17) $\dfrac{12s}{(s^2+1)(s^2+25)}$ (18) $\sqrt{\dfrac{\pi}{s}}$ (19) $\dfrac{s\cos\alpha-\omega\sin\alpha}{s^2+\omega^2}$

(20) $\dfrac{\sqrt{\pi}}{2s^{3/2}}\cdot e^{-1/4s}$ (21) $\sqrt{\dfrac{\pi}{s}}\cdot e^{-1/4s}$

TYPE II :-

(22) $\dfrac{s+b}{(s+b)^2+a^2}$ (23) $\dfrac{2}{(s-3)^2}$ (24) $\dfrac{2s-2a-3b}{(s-a)^2+b^2}$ (25) $\dfrac{\lfloor n+1}{(s+a)^{n+1}}$ (26) $\dfrac{s^2+1}{(s-1)^3}$ (27) $a\left[\dfrac{s^2-2a^2}{s^4+4a^4}\right]$

(28) $\dfrac{6}{s^2-2s+37}+\dfrac{2}{s^2-2s+5}$ (29) $\dfrac{3}{4}\sqrt{\dfrac{\pi}{s-4}}$ (30) $\dfrac{2}{(s+1)(s^2+2s+5)}$ (31) $\dfrac{s-4}{s^2-8s-9}$ (32) $\sqrt{\dfrac{\pi}{s+3}}$

(33) $\dfrac{1}{s^2+2s}$ (34) $a\left[\dfrac{s^2+2a^2}{(s^2-2as+2a^2)(s^2+2as+2a^2)}\right]$ (35) $\dfrac{1}{s^2-2s+5}$ (36) $\dfrac{\lfloor n+1}{(s+a)^{n+1}}$ (37) $\dfrac{2a^2s}{s^4+4a^4}$

(38) $\dfrac{s}{s^2-2s+5}$ (39) $\dfrac{6}{(s+2)^4}$ (40) $\dfrac{2}{(s-2)^2}$ (41) $\dfrac{2}{(s-1)^3}$ (42) $\dfrac{24}{(s+2)^4}$ (43) $\dfrac{12}{(s+3)^5}$

(44) $\dfrac{s-1}{s^2-2s+2}$ (45) $\dfrac{6}{s^2-4s+8}$ (46) $\dfrac{5(s+2)}{s^2+4s+13}$ (47) $\dfrac{4}{s^2+10s+26}$ (48) $\dfrac{1}{s-1}-\dfrac{s-1}{s^2-2s+5}$

(49) $\dfrac{1}{4}\left(\dfrac{1}{s-3}+\dfrac{s-3}{s^2-6s+13}\right)$ (50) $\dfrac{1}{s(s-2)}$ (51) $\dfrac{3(s-2)}{s^2-4s-12}$ (52) $\dfrac{6}{s^2+2s-8}$ (53) $\dfrac{s+3}{4(s^2+6s+5)}$

(54) $\dfrac{2(s-10)}{s^2-2s+10}$ (55) $\dfrac{-6(s+1)}{s(s+4)}$ (56) $\dfrac{6e^{-s}}{s^4}$ (57) $e^{-as}\cdot\dfrac{s}{s^2+1}$ (58) i) $\dfrac{a}{s}(1-e^{-bs})$

~~(59)~~ (ii) $\dfrac{1}{s^2}+e^{-4s}\left(\dfrac{1}{s}-\dfrac{1}{s^2}\right)$ (iii) $e^{-s}\left(\dfrac{1}{s}+\dfrac{1}{s^2}\right)-e^{-2s}\left(\dfrac{2}{s}+\dfrac{1}{s^2}\right)$ (iv) $\dfrac{2(1-e^{-\pi s})}{s^2+4}$ (v) $\dfrac{2e^{-s}}{s^3}$

TYPE III :-

(59) $\dfrac{s}{(s^2-a^2)^2}$ (60) $\dfrac{2s(s^2-3a^2)}{(s^2+a^2)^3}$ (61) $\dfrac{3(-s^2+4s+9)}{(s^2+9)^2}$ (62) $\dfrac{2s(s^2-3k^2)}{(s^2+k^2)^3}$ (63) $\dfrac{4(s-3)}{(s^2-6s+13)^2}$

(64) $\dfrac{2a(3s^2-a^2)}{(s^2+a^2)^3}$ (65) $-\bar{f}'(s-1)$

TYPE V :-

(66) (67) $\dfrac{1}{2}\log\left(\dfrac{s^2+a^2}{s^2}\right)$ (68) $\log\left(\dfrac{s-b}{s-a}\right)$ (69) $\dfrac{1}{2}\log\left(\dfrac{s^2+b^2}{s^2+a^2}\right)$ (70) $\dfrac{1}{2}\log\left(\dfrac{s+1}{s-1}\right)$

(71) $\cot^{-1}(s+1)$ (72) $\dfrac{1}{4}\log\left(\dfrac{s^2+4}{s^2}\right)$ (73) $\dfrac{\pi}{2}-\dfrac{s}{2}\log\left(\dfrac{s^2+1}{s^2}\right)-\tan^{-1}(s)$ (74) $\log\left(\dfrac{s-1}{s}\right)$

(75) $\log\left(\dfrac{s+b}{s+a}\right)$ (76) $\cot^{-1}\left(\dfrac{s}{2}\right)$ (77) $\dfrac{3}{4}\left(\dfrac{\pi}{3}-\tan^{-1}s+\dfrac{1}{3}\tan^{-1}\dfrac{s}{3}\right)$ (78)

PAGE 1

79) $\dfrac{4(16+12s-s^2)}{(s^2+16)^2}$ (180) $\dfrac{e^{-3/(s+1)}}{s+1}$ (81) $\dfrac{3s+8}{s^2(s+4)^{3/2}}$ (82) $\dfrac{s^2-2s+4}{4(s+1)^2(s-2)}$

(86) $\dfrac{1}{s}\cot^{-1}(s-1)$ (87) $\dfrac{1}{s}\left[\dfrac{s^2+1}{(s^2-1)^2}\right]$ (88) $\dfrac{1}{2}\left[\dfrac{s-2}{(s-1)^2(s-3)}\right]+\dfrac{1}{2}\left[\dfrac{s}{(s+1)^2(s-1)}\right]$

(89) $\dfrac{6}{s(s-1)^4}$ (90) $\dfrac{s-1}{s(s^2-2s+2)}$ (91) $\dfrac{1\times\log}{s}\left(\dfrac{s+1}{s}\right)$ (92) $\dfrac{3(3s^2+16s+25)}{s^2(s^2+8s+25)^2}$

(93) $\dfrac{6}{(s^2+6s+18)^2}$ (94) $\dfrac{4(s+4)}{s(s^2+8s+20)^2}$ (95) $\bar{y}(s)[s^2-3s+5]-2s+10$

(96) $\dfrac{1}{\sqrt{s}}$ (97) $s\cot^{-1}s-1$ (98) $s\left[\dfrac{1}{s}+\dfrac{1}{s^2}-\dfrac{e^{-2s}}{s^2}\right]-1$ (99) 7

(100) i) 3 (ii) 16 (101) i) 2 (ii) 4 (102) i) $\dfrac{s}{(s^2+a^2)^{3/2}}$ (ii) $\dfrac{1}{\sqrt{s^2+2as+2a^2}}$ (iii) 1

(iv) $\dfrac{s}{(s^2+16)^{3/2}}=\dfrac{3}{125}$ (103) i) 0 (ii) $\dfrac{6}{65}$ (iii) $\dfrac{3}{50}$ (iv) $\dfrac{1}{2}\log 3$

(v) $\dfrac{1}{2}\log\left(\dfrac{2}{3}\right)$ (vi) $\dfrac{3}{50}$ (vii) $\dfrac{4}{15}$ (viii) $\log 3$ (ix) $\dfrac{\pi}{4}$ (x) $\dfrac{1}{2}\tan^{-1}(3)-\dfrac{\pi}{8}$